A HYBRID KALMAN FILTER-FUZZY LOGIC ARCHITECTURE FOR MULTISENSOR DATA FUSION

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Abstract- In this work a novel hybrid Multi-Sensor Data Fusion (MSDF) architecture integrating Kalman filtering and fuzzy logic techniques is explored. The objective of the hybrid MSDF architecture is to obtain fused measurement data that determines the parameter being measured as precisely as possible. To reach this objective, first each measurement coming from each sensor is fed to a Fuzzy-adaptive Kalman Filter (FKF), thus there are *n* sensors and *n* FKFs working in parallel. The adaptation in each FKF is in the sense of adaptively adjusting the measurement noise covariance matrix R employing a fuzzy inference system (FIS) based on a covariance matching technique. Second, another FIS, here called a fuzzy logic observer (FLO), is monitoring the performance of each FKF. Based on the value of a variable called Degree of Matching (DoM) and the matrix R coming from each FKF, the FLO assigns a degree of confidence, a number on the interval [0, 1], to each one of the FKFs output. The degree of confidence indicates to what level each FKF output reflects the true value of the measurement. Finally, a defuzzificator obtains the fused estimated measurement based on the confidence values. To demonstrate the effectiveness and accuracy of this new hybrid MSDF architecture, an example with four noisy sensors is outlined. Different defuzzification methods are explored to select the best one for this particular application. The results show very good performance.

Index terms: Multi-sensor data fusion, Knowledge-based sensor fusion, fuzzy logic, adaptive Kalman filtering.

I. INTRODUCTION

The Multi-Sensor Data Fusion (MSDF) approach is described as the acquisition, processing, and synergistic combination of information gathered by various knowledge sources and sensors to provide a better understanding of a phenomenon under consideration [1]. Different MSDF techniques have been explored recently. These techniques vary from those based on well-established Kalman filtering methods [2], [3], to those based on recent ideas from soft computing technology [4], [5]. However, little work has been done in exploring hybrid architectures that consider both these approaches. In this work a novel MSDF architecture is explored. This architecture is based on a hybrid structure integrating fuzzy inference systems and Kalman filtering techniques.

The general idea explored here is the combination of the advantages that both techniques have. On the one hand, Kalman filtering is recognised as one of the most powerful traditional techniques of estimation. This relies on the fact that the Kalman filter is an optimal linear estimator; that means it estimates are linear, unbiased, and with minimum error variance [6]. On the other hand, the main advantages derived from the use of fuzzy logic techniques, with respect to traditional schemes, are the simplicity of the approach, the capability of fuzzy systems to deal with imprecise information, and the possibility of including heuristic knowledge about the phenomenon under consideration.

The remainder of this paper is organised as follows. Section II describes the Kalman filter. Section III summarizes the proposed fuzzy-adaptive Kalman filter; a more detailed description of this approach is given in Escamilla and Mort [7]. Section IV introduces the proposed new hybrid MSDF architecture where fuzzy logic and Kalman filtering techniques are integrated. In order to show the effectiveness of this MSDF architecture, in section V an illustrative example is outlined and results are discussed. Finally, in section VI the conclusions of this work are given.

II. THE KALMAN FILTER

The Kalman filter is an optimal recursive data processing algorithm [6] that provides a linear, unbiased, and minimum error variance estimate of the unknown state vector $x_k \in \Re^n$ at each instant k = 1, 2, ..., (indexed by the subscripts) of a discrete-time controlled process described by the linear stochastic difference equations:

$$x_{k+1} = A_k x_k + B_k u_k + w_k$$
(1)

$$z_k = H_k x_k + v_k \tag{2}$$

where x_k is an $n \times 1$ system state vector, A_k is an $n \times n$ transition matrix, u_k is an $l \times 1$ vector of the input forcing function, B_k is an $n \times l$ matrix, w_k is an $n \times 1$ process noise vector, z_k is a $m \times 1$ measurement vector, H_k is a $m \times n$ measurement matrix, and v_k is a $m \times 1$ measurement noise vector.

Both w_k and v_k are assumed to be uncorrelated zero-mean Gaussian white noise sequences with covariances,

$$E\left\{w_{k}w_{i}^{T}\right\} = \begin{cases} Q_{k}, & i=k\\ 0, & i\neq k \end{cases}$$
(3)

$$E\left\{\!\boldsymbol{v}_{k}\boldsymbol{v}_{i}^{T}\right\}\!=\begin{cases}\boldsymbol{R}_{k}, & i=k\\ 0 & i\neq k\end{cases}$$
(4)

$$E\left\{w_{k}v_{i}^{T}\right\}=0 \qquad \text{for all } k \text{ and } i \qquad (5)$$

where $E\{\cdot\}$ is the statistical expectation, superscript *T* denotes transpose, Q_k is the process noise covariance matrix, and R_k is the measurement noise covariance matrix.

The Kalman filter algorithm [8] can be organised in two groups of equations,

i) Time update (or prediction) equations:

$$\hat{x}_{k+1}^{-} = A_k \hat{x}_k + B_k u_k \tag{6}$$

$$P_{k+1}^{-} = A_k P_k A_k^{T} + Q_k$$
(7).

These equations project, from time step k to step k+1, the current state and error covariance estimates to obtain the *a priori* (indicated by the super minus) estimates for the next time step.

ii) Measurement update (or correction) equations:

$$K_{k} = P_{k}^{-} H_{k}^{T} [H_{k} P_{k}^{-} H_{k}^{T} + R_{k}]^{-1}$$
(8)

$$\hat{x}_{k} = \hat{x}_{k}^{-} + K_{k} [z_{k} - H_{k} \hat{x}_{k}^{-}]$$
(9)

$$P_{k} = [I - K_{k} H_{K}] P_{k}^{-}$$
(10).

These equations incorporate a new measurement into the *a priori* estimate to obtain an improved *a posteriori* estimate.

In the above equations, \hat{x}_k is an estimate of the system state vector x_k , and P_k is the covariance matrix corresponding to the state estimation error defined by,

$$P_{k} = E\left\{ (x_{k} - \hat{x}_{k})(x_{k} - \hat{x}_{k})^{T} \right\}$$
(11)

the term $H_k \hat{x}_k^-$ is the one-stage predicted output \hat{z}_k , and $(z_k - H_k \hat{x}_k^-)$ is the one-stage prediction error sequence, also referred to as the innovation sequence or residual, generally denoted as *r* and defined as:

$$r_{k} = (z_{k} - H_{k}\hat{x}_{k}^{-})$$
(12).

The innovation represents the additional information available to the filter as a consequence of the new observation z_k . The weighted innovation, $K_k[z_k - H_k \hat{x}_k^-]$, acts as a correction to the predicted estimate \hat{x}_k^- to form the estimation \hat{x}_k ; the weighting matrix K_k is commonly referred to as the filter gain or the Kalman gain matrix.

The Kalman filter algorithm starts with initial conditions at k = 0 being: \hat{x}_0^- , and P_0^- . With the progression of time, as new measurements z_k become available, the cycle estimation-correction of states and the corresponding error covariances can follow recursively ad infinitum.

III. THE FUZZY-ADAPTIVE KALMAN FILTER

As described previously, the traditional Kalman filter formulation assumes complete *a priori* knowledge of the process and measurement noise statistics, matrices Q and R. However, in most practical applications these statistics are initially estimated or, in fact, are unknown. The problem here is that the optimality of the estimation algorithm in the Kalman filter setting is closely connected to the quality of these *a priori* noise statistics [9], [10]. It has been shown how poor estimates of the input noise statistics may seriously degrade the Kalman filter performance, and even provoke the divergence of the filter [11]. From this point of view it can be expected that an adaptive formulation of the Kalman filter will result in a better performance or will prevent filter divergence.

In next section, an on-line adaptive scheme of the Kalman filter employing the principles of fuzzy logic is presented. The adaptation is in the sense of adaptively adjusting the measurement noise covariance matrix R from data as they are obtained. The main advantages derived from the use of fuzzy techniques, with respect to traditional adaptation schemes, are the simplicity of the approach, the possibility of including heuristic knowledge about the phenomenon under consideration, and the relaxation of the a priori statistical assumptions.

A. Adaptive estimation of the measurement noise covariance matrix R

The covariance matrix R represents the accuracy of the measurement instrument. A larger value of the covariance matrix R for measured data means that we trust this measured data less and have more faith in the prediction. Assuming that the noise covariance matrix Q is completely known, an algorithm to estimate the measurement noise covariance matrix R can be derived.

Here an innovation-based adaptive estimation (IAE) algorithm [12] to adapt the measurement noise covariance matrix R has been derived [7]. In particular, the technique known as covariance-matching [13] is used. The basic idea behind this technique is to make the actual value of the covariance of the residual consistent with its theoretical value. The innovation sequence or residual r_k has a theoretical covariance,

$$S_k = H_k P_k^- H_k^T + R_k \tag{13}$$

obtained from the Kalman filter algorithm. Given the availability of the innovation sequence r_k , its actual covariance \hat{C}_{rk} is approximated by its sample covariance through averaging inside a moving estimation window of size N [12],

$$\hat{C}_{rk} = \frac{1}{N} \sum_{i=i_0}^{N} r_i r_i^T$$
(14)

where $i_0 = k - N + 1$ is the first sample inside the estimation window. The window size *N* is chosen empirically to give some statistical smoothing.

Thus, if it is found that the actual covariance of r_k has a discrepancy with its theoretical value, then adjustments have to be made to R in order to correct this mismatch.

Now, to detect the discrepancy of S_k and its actual value \hat{C}_r a new variable is defined. This variable is called the Degree of Matching (*DoM*),

$$DoM_k = S_k - \hat{C}_{rk} \tag{15}.$$

The basic idea used by a Fuzzy Inference System (FIS) to adapt R is as follows. It can be noted from equation (13) that an increment in R will increment S, and vice versa. Thus, R can be used to vary S in accordance with the value of *DoM* in order to reduce the discrepancies between S and \hat{Q}

 \hat{C}_r . From here, three general rules of adaptation are defined:

- 1. If $DoM \cong 0$ (this means *S* and \hat{C}_r match almost perfectly) then maintain *R* unchanged.
- 2. If DoM > 0 (this means *S* is greater than its actual value \hat{C}_r) then decrease *R*.
- 3. If DoM < 0 (this means *S* is smaller than its actual value \hat{C}_r) then increase *R*.

Thus *R* is adjusted in this way:

$$\boldsymbol{R}_{k} = \boldsymbol{R}_{k-1} + \Delta \boldsymbol{R}_{k} \tag{16}.$$

where ΔR is the factor that is added or subtracted from *R* each instant of time. ΔR is the FIS output and *DoM* is the FIS input. A graphical representation of the Fuzzy-adaptive Kalman Filter (FKF) is shown in Fig. 1.



Fig. 1. Basic structure of the Fuzzy-adaptive Kalman Filter.

IV. HYBRID ARCHITECTURE FOR MULTI-SENSOR DATA FUSION

The objective of the proposed hybrid multi-sensor data fusion (MSDF) architecture is to obtain fused measurement data that determines the parameter being measured as precisely as possible. To reach this objective, it is assumed that there are n different sensors measuring the same parameter and each sensor has its own characteristics of noise and measurement errors. First, the measurements coming from these sensors are fed to a FKF, one for each sensor, thus there are n sensors and n FKFs working in parallel (see Fig. 2).



Fig. 2. Proposed MSDF architecture.

Second, another fuzzy inference system, here called the fuzzy logic observer (FLO), is used to monitor the performance of each FKF. Based on the values of the variables DoM and R coming from each FKF, the FLO assigns a degree of confidence w, a number on the interval [0, 1], to each one of the FKFs output. The degree of confidence indicates to what level each FKF output reflects the true value of the measurement. At the same time, the

degree of confidence acts as a weight that tells a defuzzificator at what confidence level it should take each FKF output value.

Finally, a defuzzificator obtains the fused estimate of the measurement based on the confidence values. Here several defuzzification procedures can be used. Fig. 2 shows a graphical representation of the proposed MSDF architecture.

V. ILLUSTRATIVE EXAMPLE

To demonstrate the effectiveness and accuracy of this new hybrid MSDF architecture, an example with four noisy sensors is outlined.

Consider the following linear system, which is a modified version of a tracking model [14], [15],

$$\begin{bmatrix} x_{k+1}^{1} \\ x_{k+1}^{2} \\ x_{k+1}^{3} \end{bmatrix} = \begin{bmatrix} 0.77 & 0.20 & 0.00 \\ 0.25 & 0.75 & 0.25 \\ 0.05 & 0.00 & 0.75 \end{bmatrix} \begin{bmatrix} x_{k}^{1} \\ x_{k}^{2} \\ x_{k}^{3} \end{bmatrix} + \begin{bmatrix} w_{k}^{1} \\ w_{k}^{2} \\ w_{k}^{3} \end{bmatrix}$$
(17a)
$$z_{k} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{k}^{1} \\ x_{k}^{2} \\ x_{k}^{3} \\ x_{k}^{3} \end{bmatrix} + v_{k}$$
(17b)



Fig. 3. (a) Noise on sensor 1; (b) noise on sensor 2; (c) noise on sensor 3; (d) noise on sensor 4.

with initial conditions $\hat{x}_0 = 0$, $P_0 = 0.01 I_3$, where x^l , x^2 , and x^3 are the position, velocity and acceleration, respectively, of a flying object. In equation (17), the system noise sequence $\{w_k\}$ is a pseudorandom sequence (i.e., uncorrelated zero-mean Gaussian white noise sequence) with $Q = 0.02I_3$.

MATLAB code was developed to simulate the process described by equation (17) and the proposed MSDF system considering four sensors measuring the position of the flying object. The simulation was carried out for 500s with a sample time of 0.5s. Q was fixed as $0.02I_3$. The actual value of R for each sensor has been assumed unknown, but its starting value in all sensors was selected as 1. The sensor measurements are corrupted with the noises described in Fig. 3. Sensor 1 is corrupted with noise 1, sensor 2 with noise 2, and so on.

In subsequent sections, the implementation of each one of the components of the hybrid MSDF architecture is described.

A. Fuzzy-adaptive Kalman Filter (FKF)

Following the general guidelines given in section III.A, each FIS used in each FKF to adjusts *R* was implemented considering three fuzzy sets for *DoM*: N = Negative, ZE = Zero, and P = Positive; and three fuzzy sets for ΔR : I = Increase, M = Maintain, and D = Decrease. These membership functions are presented in Fig. 4. Hence, three fuzzy rules are included in each FIS rule base:



Thus, using the compositional rule of inference sum-prod and the center of area defuzzification method R is adjusted in each FKF as mentioned in equation (16). The size N of the moving window in equation (14) was selected from experimentation as 15.

B. Fuzzy Logic Observer (FLO)

Each FLO was implemented using two inputs, the absolute value of DoM (*AbsDoM*) and the current value of R; and one output, the degree of confidence denoted as w. The

membership functions for *AbsDoM* and *R* are shown in Fig. 5. Here the fuzzy labels mean: ZE = zero, S = small, and L = large. For the output *w*, 3 fuzzy singletons were defined with the labels: G=1=good, AV=0.5=average, and P=0=poor. Thus 9 rules complete the fuzzy rule base of each FLO, and these are:

- 1. If AbsDoM = ZE and R = ZE, then w = G
- 2. If AbsDoM = ZE and R = S, then w = G
- 3. If AbsDoM = ZE and R = L, then w = AV
- 4. If AbsDoM = S and R = ZE, then w = G
- 5. If AbsDoM = S and R = S, then w = AV
- 6. If AbsDoM = S and R = L, then w = P
- 7. If AbsDoM = L and R = ZE, then w = AV
- 8. If AbsDoM = L and R = S, then w = P
- 9. If AbsDoM = L and R = L, then w = P.

The above rules are based on two simple heuristic considerations. First, if the absolute value of DoM is near to zero and R is near to zero then it means the filter is working almost perfectly. Second, if one or both of these values increases far from zero that means the filter performance is degrading. Thus, using the compositional rule of inference sum-prod and the centre of area defuzzification method each FLO obtains the degrees of confidence for each FKF.



Fig. 5. Membership functions for *AbsDoM* and *R*.

C. Defuzzification

Different defuzzification methods were explored to select the best one for this particular application. The results obtained with the centre of area (COA) and a variation of the maximum are reported here. In the COA method the fused measurement output \hat{z}_k is obtained as,

$$\hat{z}_{k} = \frac{\sum_{i=1}^{4} \hat{z}_{ki} w_{ki}}{\sum_{i=1}^{4} w_{ki}}$$
(18)

where \hat{z}_{ki} is the output of the *i*-th FKF (*i*=1,2,3,4) and w_{ki} is its respective degree of confidence at instant of time *k* (see Fig. 2). Strictly speaking, in this case the COA method is simply a weighted average. That is, each FKF output is weighted according to its corresponding degree of confidence, *w*. In the maximum method the fused measurement output is that one which corresponds to the

FKF output that has the maximum degree of confidence at each instant of time k.

In order to prevent possible conflicts, one modification for each method was incorporated. For the COA method, if the sum of the degrees of confidence is equal to zero, then the fused output is simply the average of the FKF outputs. For the case of the maximum, if there is more than one maximal degree of confidence, then the FKF output corresponding to the first maximum encountered is given as the fused measurement.

D. Results

For comparison purposes, the following performance measures were adopted:

$$J_{zv} = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (za_k - z_k)^2}$$
(19)

$$J_{ze} = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (za_k - \hat{z}_k)^2}$$
(20),

where za_k is the actual value of the position; z_k is the measured position; and \hat{z}_k is the estimated position at an instant of time k; n = No. of samples.



Fig. 6. (a) Actual and fused estimated position obtained with the MSDF architecture using the COA defuzzification method. (b) Corresponding error on the fused estimated position.

Table 1 shows the performance measures obtained for each individual FKF and those obtained from the fusion of the four sensors using the proposed MSDF architecture with both defuzzification methods mentioned above. Analysing the data, it is noted that the best estimated position is obtained with the MSDF architecture using the COA defuzzification method. In this case the error on the estimation is 17% less with respect to that obtained with

FKF 4 (for sensor 4), which has the best individual performance measure. At the same time this error is 52% less with respect to that obtained with FKF 2 (for sensor 2) which has the worst individual performance measure. Fig. 6a shows the actual and fused estimated position; and Fig. 6b shows the corresponding error on the estimation, for this case.

For the case of the MSDF using the maximum defuzzification method, the performance measure is only a little larger than that for the case previously analysed. However, this performance measure shows smaller error estimation than that observed in the best individual FKF (number 4). Thus both fused measurements are more exact than any individual FKF estimation.

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Sensor	J_{zv}	J_{ze}
Sensor 1	2.0604	0.4180
Sensor 2	3.3701	0.5994
Sensor 3	1.9224	0.3719
Sensor 4	1.9446	0.3454
Fused – COA		0.2866
Fused – Maximum		0.3011

VI. CONCLUSIONS

A novel hybrid MSDF architecture integrating Kalman filtering and fuzzy logic techniques has been presented. This approach exploits the advantages that both approaches have: the optimality of the Kalman filter and the capability of fuzzy systems to deal with imprecise information using 'common sense' fuzzy rules.

In this novel approach the linear estimations of the individual Kalman filters are improved through the adaptation of the measurement noise covariance matrix R by means of a FIS. This prevents filter divergence and relaxes the a priori assumption of the value of R. It is particularly relevant that only three rules were needed to carry out the adaptation.

The role of the FLO in the proposed MSDF architecture is of great importance. This is because the fusion of the information is carried out based on the degrees of confidence generated on this component no matter what defuzzification method is used. Other important points are that only two variables were monitored to establish the degree of confidence for each FKF output and only nine 'common sense' fuzzy rules were needed.

The results obtained in the illustrative example are promising. They show that this novel hybrid MSDF architecture is effective in situations where there are several sensors measuring the same parameter and each sensor measurement is contaminated with a different kind of noise. Both fused estimated measurements (with COA and maximum defuzzification methods) were better approximations to the actual value of the parameter being measured than that obtained with any single FKF.

Thus the general idea of exploring the combination of traditional together with non-traditional techniques appears to be a good avenue of investigation.

The system employed to illustrate the effectiveness of the approach presented was very simple and only one parameter is considered as being measured. However, the approach can be easily extended for systems with more than one parameter being measured. In fact, this is the subject of current work by the authors.

The choice of the fuzzy sets used in the FISs was carried out using a trial and error scheme. Obviously this process is time consuming and depends on the problem under consideration. In order to tackle this problem the authors are exploring the idea of using a neuro-fuzzy system to adjust automatically these fuzzy sets. For the case of the fuzzy rules, the general guidelines given for both cases (FKF and FLO) showed its effectiveness in the chosen example.

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