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Engineering Notes

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Soft-Histogram Degradation Analysis of a Tie Bar of a Rotor-Head Structure

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I. Introduction

IN THIS work, a waveform-based feature-extraction algorithm, referred to as a soft histogram feature-extraction algorithm (SH-FEA), is developed to extract damage-sensitive information from measured-response data of a tie-bar component of the main rotor hub of a Lynx helicopter. The feature-extraction algorithm is based on fuzzy-sets theory. The results of applying the proposed feature-extraction approach to analyze tie-bar data to reveal the mechanism toward failure of this component are presented.

II. Soft Histogram Feature-Extraction Algorithm

A. Hard Histogram

Let us define x as a random variable [e.g., a signal $f(x)$] from which N number of samples x_i ($i = 1, \ldots, N$), have been obtained (e.g., measured). A hard histogram is that in which the N samples x_i are separated into a set of M equal-width disjoint *bins*. Each bin accumulates the total number of samples in the signal that have a value within a certain range. Thus, if the bin centers are denoted as c_k and the bin width (and bin distance) is represented as d, then the histogram value for bin number k ($k = 1, \ldots, M$) can be written as

$$h_k(x) = \sum_{i=1}^{N} H_k(x_i) \quad \text{with} \quad H_k(x_i) = \begin{cases} 1 & \text{if } |x_i - c_k| < d/2 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $H_k(x_i)$ is the activation function for bin k, which also can be seen as a crisp rectangular membership function. Hard histograms provide the information about the dynamic range of the values of a signal. Traditionally, hard histograms are used to calculate the first-order statistics of the analyzed signal and to approximate its probability density function [1].

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B. Soft Histogram

A soft histogram (also known as a fuzzy histogram [2]) is a histogram with partially overlapping and smooth bins, referred to as *fuzzy bins*. In a soft histogram, instead of letting each sample fall into one of the bins, they are allowed to fall into two or more neighboring bins, but only partially. Using the same notation as in Eq. (1), the histogram value for fuzzy-bin number k ($k = 1, \ldots, M$) is written as

$$h_k(x) = \sum_{i=1}^{N} \mu_k(x_i) \quad \text{with}$$
$$\mu_k(x_i) = \begin{cases} \max(0, 1 - \frac{|x_i - c_k|}{d}) & \text{if } |x_i - c_k| \leq d \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where $\mu_k(x_i)$ is the activation function for fuzzy bin k. In this case, the kth fuzzy bin is characterized by the membership functions $\mu_k(x_i)$, which is a symmetrical triangular fuzzy set with center c_k and width d. This implies that an overlapping of 50% exists between neighboring bins. Note that $h_k(x)$ in Eq. (2) is the same as in Eq. (1), apart from the fact that instead of accumulating frequencies (ones), the fuzzy bins accumulate grades of membership (values in the range of 0–1). Although triangular membership functions are proposed for the fuzzy bins in this work, other types of membership functions could be used (e.g., Gaussian fuzzy sets).

C. Feature-Extraction Algorithm

Let us denote the signal under consideration as $f(x)$. Then the SH-FEA consists of first normalizing the incoming signal to the range of -1 to 1 and then obtaining its soft histogram. The normalization is achieved by applying the following relationship:

$$\tilde{f}(x) = \frac{f(x)}{\max(abs(f(x))} \tag{3}$$

where $\tilde{f}(x)$ denotes the normalized signal. The soft histogram is calculated as is indicated in Sec. II.B. The M-obtained histogram values are organized in the vector **FV**, referred to as the feature vector:

$$\mathbf{FV}(x) = [\, h_1(x) \quad \cdots \quad h_M(x)\,]^T \tag{4}$$

The feature vector is the output of the proposed SH-FEA. The feature vector is used to represent the characteristics of the analyzed signal and can be used for applications of pattern recognition.

III. Degradation Analysis of the Tie Bar

Current helicopter rotors spin at near-constant revolutions per minute throughout a flight mission [3]. Consequently, it is assumed that signals coming from monitored components of the main rotor hub will be periodic signals, for which the basic frequency is the rotor frequency. With this in mind, the general idea explored to analyze the mechanism of degradation in rotor-head components is as follows: If a feature vector can be extracted that represents the characteristics of a cycle or a series of cycles of the measured signal from the component being monitored, then this feature vector can be used to perform comparisons with a cycle or a series of cycles of the signal obtained over different periods of time. This in turns will make it possible to assess how the signal is evolving until failure on the monitored component is reached, and this can be used for pattern recognition and component critical-degradation detection before the point of failure. In other words, the main postulate is that the change of the dynamic behavior of the system being monitored can be expressed in terms of changes in the feature vectors extracted from every cycle (or series of cycles) of the measured signals and compared over time.

In this section, feature-extraction results are presented that correspond to the SH-FEA applied to extract damage-sensitive information from the measured-response data of tie-bar components of the main rotor hub of a Lynx helicopter. The tie bar is the connection element between the rotor hub and the blade (see Fig. 1). The results presented correspond to the analysis of data gathered in tests carried out in a purpose-built test rig. Several tie bars were subjected to a high-level ground–air–ground (GAG) cyclic load testing. In each test, two tie bars were installed back to back in the test rig. Then cyclic twist and axial loads were applied to the tie bars, simulating the loading regime on the aircraft and tested until one of the tie bars failed. The axial load (kN), the angle of twist (deg), and the two tie-bar (referred to as TB1 and TB2, respectively) extension displacements (mm) were the parameters measured, constantly monitored, and recorded on a digital chart recorder and stored in computer files. From here on, these four signals are referred to as axial load, twist, TB1 displacements, and TB2 displacements, respectively. For brevity, results are presented for only one of the tests carried out. It is worth noting that similar results were obtained in five other additional tests.

Feature vectors were extracted using the SH-FEA for every cycle of the 4 measured signals for the test under consideration, which consisted of 1296 cycles of GAG load testing until tie bar 1 (TB1) failed. Six fuzzy bins (sets) were defined to obtain the soft histograms, with centers at -1, -0.6, -0.2, 0.2, 0.6, and 1 and with a width of 0.4 (see Sec. II.B). Examples of the normalized signals obtained at cycle 11 and corresponding extracted feature vectors (soft histograms) are shown in Fig. 2. The feature vectors obtained for every cycle of every signal are shown in Fig. 3. Note that the numbers in the horizontal axis in each plot correspond to the number of fuzzy bins, and the feature-vector values (soft histograms) obtained for every cycle of the test are represented as bars and are associated with the vertical axis.

With the extracted feature vectors available, a reference feature vector was calculated by averaging the feature vectors corresponding to cycles 11 to 60. The first 10 cycles were considered as a test warming-up process. The reference feature vectors for each signal represent the signature of the tie bars when they are in a healthy state. A comparison analysis was performed by calculating the angle between the reference feature vector and the remaining feature vectors. Recall that the angle between two vectors **x** and **y** is defined as

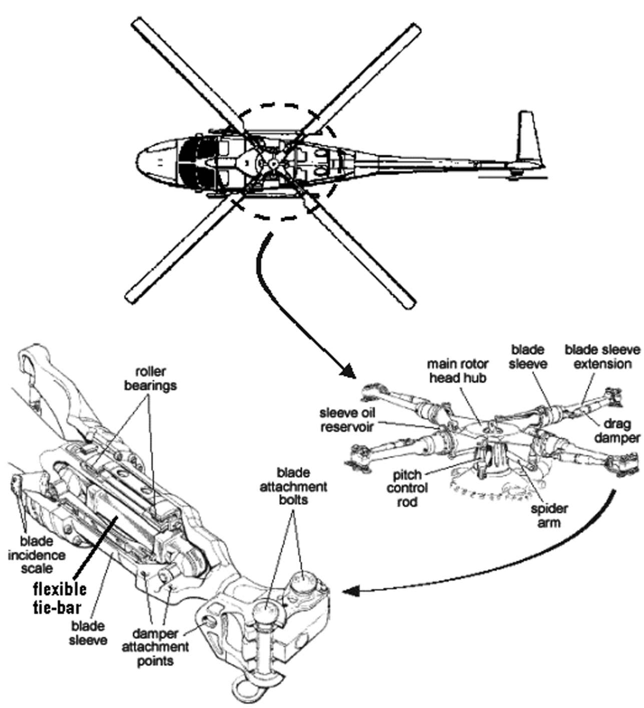


Fig. 1 Tie-bar location.

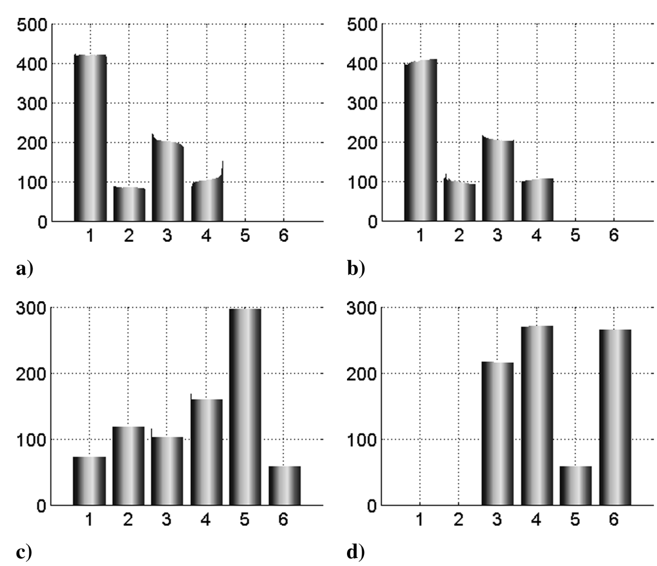


Fig. 3 Feature-vector history for a) signal TB1 displacements, b) signal TB2 displacements, c) signal twist, and d) signal axial load.

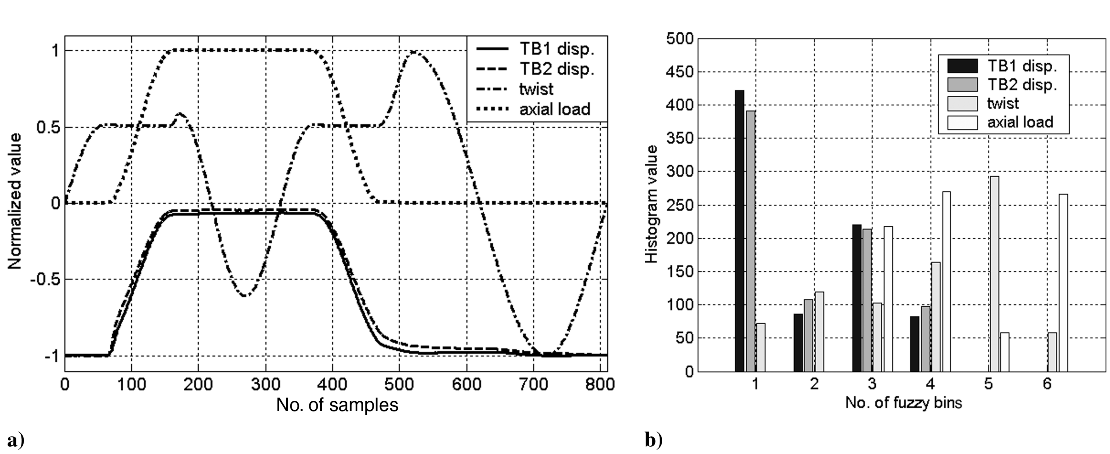
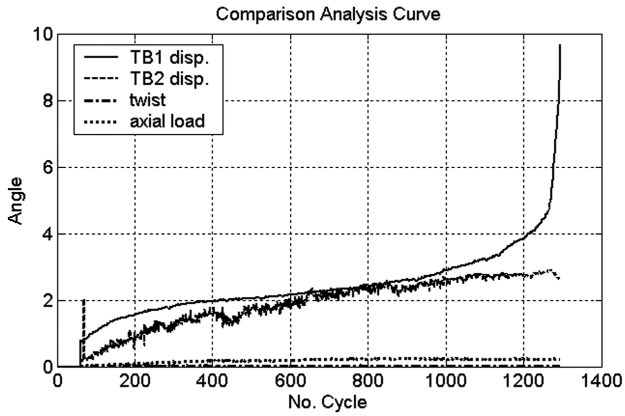
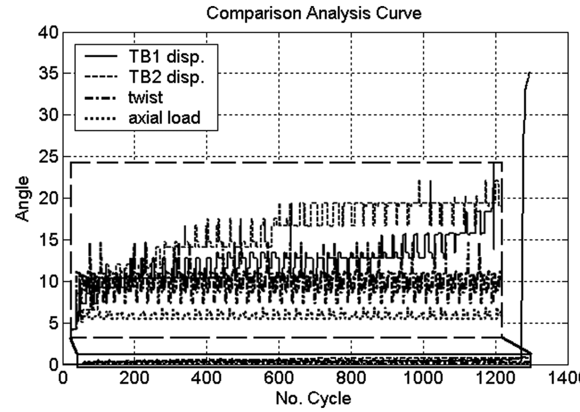


Fig. 2 Plots of a) normalized measured signals and b) corresponding feature vectors.



a)



b)

Fig. 4 Comparison-analysis curves a) using a soft histogram and b) using a hard histogram.

$$\theta(\mathbf{x},\mathbf{y}) = \cos^{-1}\left(\frac{(\mathbf{y}^{\mathrm{T}}\mathbf{x})}{\|\mathbf{x}\|\ \|\mathbf{y}\|}\right) \tag{6}$$

It is expected that the dynamic behavior of the tie bars before the point of failure can be expressed in terms of the variation of the extracted feature vectors over time when compared with the corresponding reference feature vectors. The plot of the number of cycle versus the angle between the reference feature vector and remaining feature vectors is referred to as a comparison-analysis curve. Figure 4a shows the comparison-analysis curves obtained for each one of the four measured signals.

From the comparison-analysis curves, it is clear that a pattern emerges in the data corresponding to the displacement signal of the tie bar that has failed: TB1. From this pattern, it can be said that the mechanism of degradation of tie bars when approaching failure takes place in three stages. In the first stage, degradation occurs rapidly during the first cycles. In the second stage, degradation shows a relatively slow and steady growth rate. In the third stage, degradation grows rapidly, resembling an exponential function, before the tie bar fails.

On the other hand, the comparison-analysis curve corresponding to signal TB2 displacements resembles a linear increasing function, with near-constant slope, indicating constant degradation of the tie bar but without approaching failure. Note also in Fig. 4a that the angle-comparison curves corresponding to the twist and axial loads resemble quasi-constant functions, indicating that these cyclic loads remain constant over time. These observations also can be deduced from the histogram histories shown in Fig. 3.

To compare results with those obtained using a hard histogram, instead of a soft histogram in the feature-extraction algorithm, Fig. 4b plots the comparison-analysis curves obtained using a hard histogram with 6 bins. Note that it is difficult to distinguish any pattern in these curves in this case. Only a steep change can be appreciated in TB1 displacements, just before the corresponding tie bar fails. This would not be useful to detect critical degradation of tie bars with good anticipation of the point of failure.

IV. Conclusions

In this work, a feature-extraction algorithm to extract damage-sensitive features of a tie bar component of the main rotor hub of a Lynx helicopter has been developed. The algorithm is based on the concept of a soft histogram. When applied to actual data obtained from cyclic load testing of tie bars, the soft-histogram-based feature-extraction algorithm has demonstrated to be effective in extracting damage-sensitive features, as can be seen in the pattern of the comparison-analysis curve. It is worth noting that this pattern resembles the fatigue-damage-accumulation curve observed in composite materials [4].

Acknowledgments

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