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SHAPE DESCRIPTION AND SHAPE SIMILARITY MEASUREMENT FOR TWO-DIMENSIONAL REGIONS

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ABSTRACT

Bribiesca, E. and Guzmán, A., 1979. Shape description and shape similarity measurement for two-dimensional regions. Geo-Processing, 1: 129 - 144

The study of shape is an important part of image processing and understanding. This article explains how to represent the shape or form of flat regions limited by simply connected curves. This description produces for every region (through an algorithm explained here) a shape number, which is unique and independent of the translation, rotation and scaling of the region.

The precision in the representation obtained through a shape number is indicated by the order of that shape number. The higher the order, the more accurate the shape description. Informally, the number of ternary digits in a shape number tells its order.

The paper contains tables of all the shape numbers of order K, for several K. Nevertheless, these tables are not necessary for computing the shape number of a region.

The shape number of any order can be deduced exclusively from the region; no shape matching, table lookup or string comparison is necessary.

The article then introduces the degree of similarity between the shapes of two regions and gives an algorithm for computing it from the corresponding shape numbers. Two regions with shapes that look alike will have a high degree of similarity.

No string matching or grammatical parsing is necessary to find out how close in shape two regions are. Informally speaking, the degree of similarity between the shapes of two regions is the highest optical resolution (power of the magnifying lens) that still confuses them.

Finally, the paper defines the distance between two shapes and find it to be an ultradistance.

In this way, a quantitative study of shape is possible.

At the end of the paper, a related Theory "B" of shapes is presented that disregards the eccentricity of a region and offers additional advantages for shape comparison.

INTRODUCTION

Scene Analisis seeks to understand a scene, for instance by assigning names to its different parts and components as well as by explaining their relations and structures.

Local and global information (Guzmán,1971), that is, shape and context, play an important and mutually supporting role in Scene Analysis. If we look at scenes found in coloring books for children (Fig. 1), the explanation (name, purpose, role) of each part is derived both from its shape and from the context, that is to say, from the names of the parts close to it.

The role of shape in Scene Analysis

Take Fig. 1 which lacks color, texture, gray levels, and only has shapes, sizes and structure. One can still make a good "explanation" and understanding of it. Consequently, one of the authors has proposed (Guzmán,1971) to represent explicitly these three components, for instance by a graph where the nodes contain shape and size information about each region, and the arcs represent different relations ("above","between", "surrounded by") among the nodes.

It is therefore necessary to be able to describe the shape (Pavlidis,1978) of an object (part, region); to compare shapes; to decide how close two given shapes are, or what is their resemblance or dissimilarity. A numerical reliable measure for these concepts will give rise to a quantitative study of shape.

Definitions

Region. A simply connected portion of a plane limited by a curve boundary. That is, no holes, no self-intersecting boundary. Closed boundary. The region is uniquely defined by the curve it has as boundary.

This paper deals with shapes of regions, but the shape numbers used here can also be applied to open curves.

Freeman chain in four directions. For a given region and a given square grid of fixed orientation and size, the Freeman chain in four directions is the curve obtained by walking clockwise on the grid (on the "wires" of the grid) around and outside the squares that contain more than 50% of the region (Fig. 2).

The chain number (Fig. 2d) is obtained by clockwisely replacing each step along the curve by the number 1, 2, 3, or 4, according to Fig. 2e. See suggestions 1 and 2 at the end of the paper.

Sometimes this procedure will break thin portions of regions and one will end up with two non-connected chains. These are degenerate regions for that grid, which have no shape numbers (q. v.) (Fig. 8.II).

Derivative of Freeman chains. It is the chain number (Fig. 2f) obtained by clockwise replacing each salient (convex) corner of the Freeman chain (Fig. 2c) by a 1, each straight corner by a 2, and each concave corner by a 3, as Fig. 3g suggests.

The number obtained (Fig. 2f) will be different if we change the size or orientation of the grid.

Major axis of a region. The straight line connecting the two perimeter points furthest away from each other (Fig. 3b).

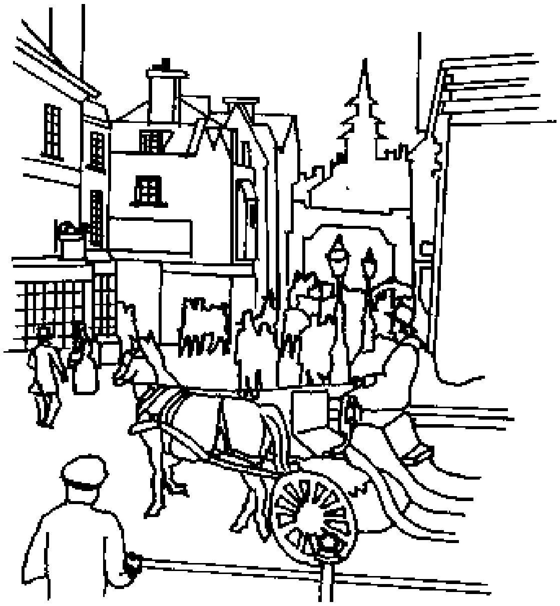

FIGURE I " STREET SCENE "

Each closed part (region) of this scene could be described by a unique shape number.

Ocassionally, there will be more than one major axis in a region. In that case, select that which gives the shortest minor axis.

<u>Minor axis</u> of a region. A segment perpendicular to the major axis, and of length and position such that the box formed by these two axes just encloses the region (Fig. 3a).

Other axis for similar purposes are given by Guzmân (1976,pp. 338-342), and by Freeman and Shapira (1975).

<u>Basic rectangle</u> of a region. It is the rectangle having its sides parallel to and of sizes equal to the major and minor axis, such that it just encloses the region (Fig. 3d).

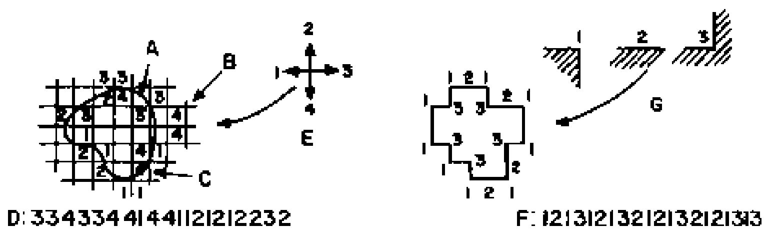

FIGURE 2 " CHAINS "

A: the region. B: the grid. C: the Freeman chain in four directions. D: its chain number. E: the four directions of (B) used to code (C) into (D). F: the derivative of (C). G: the three types of corners used to code (C) into (F).

Eccentricity of a rectangle. It is the ratio of the long to the short side. $e \geq 1$.

Eccentricity of a region. It is the eccentricity of its basic rectangle. It is the ratio of its major to minor axis. This definition coincides with that for an ellipse.

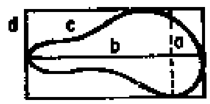

FIGURE 3 " DEFINITIONS "
a: minor axis of (c). b: major axis of (c). c: region.
d: basic rectangle of (c).

THE SHAPE NUMBER OF A REGION

If a notation is going to be used to represent the shape of a region, it has to be independent of the position, orientation and size of such region. It should be reproducible: a region, when translated, magnified and rotated should still give the same description as when untransformed. Two regions with different shapes should produce different descriptions. Finally, the shape number should be unique for a given region; for instance, it should not depend on an arbitrary starting point or a particular coordinate system.

If the notation can be deduced exclusively from the region, without comparison with a table of canonical shapes or shape descriptors, for instance, then we can expect savings in memory and computer time for the procedure that finds out the shape description.

In this section, we first produce finite families of shape descriptors (every member of a family has the same order); we then exhibit a way to find out, for an arbitrary region, its shape descriptor of any order. This descriptor indeed qualifies as a notation to represent shape.

In the next section we will see that this descriptor also permits to measure the similarity or analogy between the shapes of two regions.

Discrete shapes

Regions of special interest are created when it is required to form a closed curve using o sticks of the same length, but joining them end to end either colinearly or forming 90° corners. It is clear that o must be even for the curve to close.

For instance, with 8 sticks you could form only the following regions: the square (of size 2 by 2, Fig. 4a), the triangle (Fig. 4b) and the rectangle (Fig. 4c).

The shapes of these regions are called discrete shapes.

The shape number of a discrete region (that is, of a region having a discrete shape) is obtained from that region by clockwisely replacing each salient corner by a 1, each straight corner by a 2, and each concave corner by a 3 (Fig. 2g). Moreover, in order to obtain a unique shape number, we start the procedure in the corner that produces a string (of 1, 2 and 3's) of minimum value.

For instance, the shape number of Fig. 4b is 11212113, which was obtained by starting in the upper central salient corner and travelling clockwise (first right and then down). Had we started in the lower left corner, we would have obtained 11311212 which is rejected because its value (as a ternary number) is larger than 11212113.

The shape number of a discrete shape does not depend on a grid of fixed orientation or size; it can be derived directly from the region. It differs in this manner from "derivative of Freeman chain".

The shape number of a discrete shape is unique. It does not depend on its position, size or orientation.

The order of a shape number is the number of ternary digits it has. It is therefore equal to the number of corners (of types 1, 2 and 3 in Fig. 2g) that the discrete shape has.

It is also the number of sticks (segments of equal length) present in the discrete shape. It is always even. It is equal to the perimeter of the region.

All the discrete shapes of order 4

There is only one discrete shape of order four, the square. Its shape number is 1111.

This is the most primitive or fundamental shape. Imagine you are looking at things very far away; you can not really differentiate much. All objects would look round (square, in this paper) and equal.

All the discrete shapes of order 6, 8, 10 and 12

There is only one discrete shape of order 6, the rectangle with shape number 112112.

The three discrete shapes of order 8 are given in Fig. 4. Here the triangle appears for the first time.

The nine discrete shapes of order 10 are given in Fig. 5; those of order twelve are in Fig. 6. They are 36.

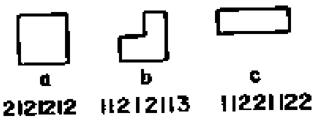

FIGURE 4 " ALL THE DISCRETE SHAPES OF ORDER 8 "

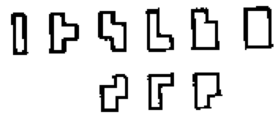

FIGURE 5 " ALL THE DISCRETE SHAPES OF ORDER 10 "

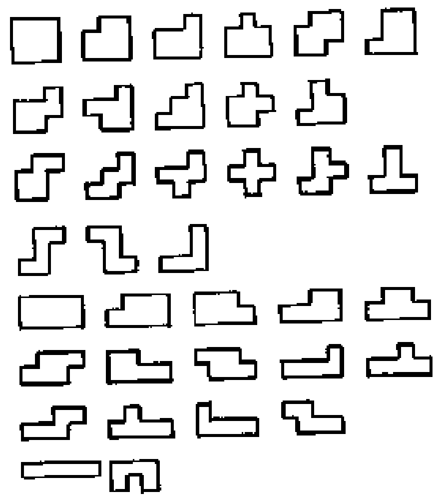

FIGURE 6 " ALL THE DISCRETE SHAPES OF ORDER 12 "
The order of a shape indicates the number of sticks that are used to form it.

The discrete shape of a region

In order to find out the shape number of order o for an arbitrary region (and not just for the regions having discrete shapes), it is now only necessary to associate in some manner to that region a discrete shape, and then to give the shape number of that discrete shape to the region itself.

One way to proceed would be to compare (for instance, the areas in the least squares sense) that region with every discrete shape of order o (retrieved from a table such as Fig. 5)[1] and to select the discrete shape having the best fit (smallest error, best correlation).

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[1]This method could produce different shape numbers than those found by the procedure described in the paper. Both methods are not equivalent.

Other way is given below, preferred because it does not use table lookup, backtracking, error computation or pattern matching in the CONVERT (Guzmán and McIntosh, 1966) sense: we do not need to find out what is the distance or error between 11212113 and 12121212, for instance.

<u>To find the shape number of order o of a region:</u>

1. Find out the basic rectangle and the eccentricity of the region.
2. Select the rectangle with shape number o and eccentricity closest to e. Align and center this rectangle over the basic rectangle of the region, thus defining a grid over the region.

   The orientation of the grid follows the basic rectangle, and the size of the grid is such that (a) every cell of the grid is a square, and (b) the basic rectangle has a shape number of order o for such grid. Already positioned, the rectangle selected in this step closely coincides with the basic rectangle.

   In practice, we have found better not to approximate the eccentricity, but the sides of the rectangle instead. That is, select a rectangle with long side closest to $y=(o/2)(e/1+e)$.
3. Mark with a 1 each cell of the grid of step 2 that is more than 50% contained in the region.

   The collection of grid squared containning a 1 forms a discrete shape.
4. Find the shape number of the discrete shape of step 3, and give that as answer (but see discussion below).

An example is given in Fig. 7.

Is the shape number found in step 4 indeed of order o? The crucial step is 3 above. The answer is discussed after an alternative step 3.

3bis (variant). On the perimeter of the square selected in step 2, place o sticks on the "wires" of the grid. Looking at each corner (of type 1 in Fig. 2g) of these sticks, push it and make it become a corner of type 3 if it surrounds a cell of the grill filled less than 50% with the region. Keep pushing corners (Fig. 7, steps 3bis) until no further progress is possible. (Then go to step 4 above). This step 3b could be taken instead of step 3.

It is clear that this step does not alter the order of the shape number, since the number of sticks does not change.

What could increase the number of sticks (the length of the perimeter) is a depression in the boundary, because (Fig. 8) in order to sink stick a to position b we need two extra sticks. In this case we end up with a shape number of order o+2, or in general of order o + 2d, where d is the depth of the depression.

The way to correct this anomaly is to begin step 2 by selecting a rectangle not of order o (because we have just found that o produces a shape number of order o + 2d) but of order o - 2d, and then the depression will add 2d sticks to it, obtaining a shape number of the correct order.

Since a depression changes depth as the size of the grid varies, we may have to try step 2 with rectangles of order o-2d, o-2d+2, ..., o-2, until we find the shape number of order o.

Informally speaking, the order of the shape number is the degree of resolution being used to encode the shape.

The eccentricity of the shape is important. It is a shape parameter coarser than the shape number. Two shapes of order o with basic rectangles of different eccentricities can not be equal. The basic rectangle and the eccentricity can be directly computed from the shape numbers (suggestion 6).

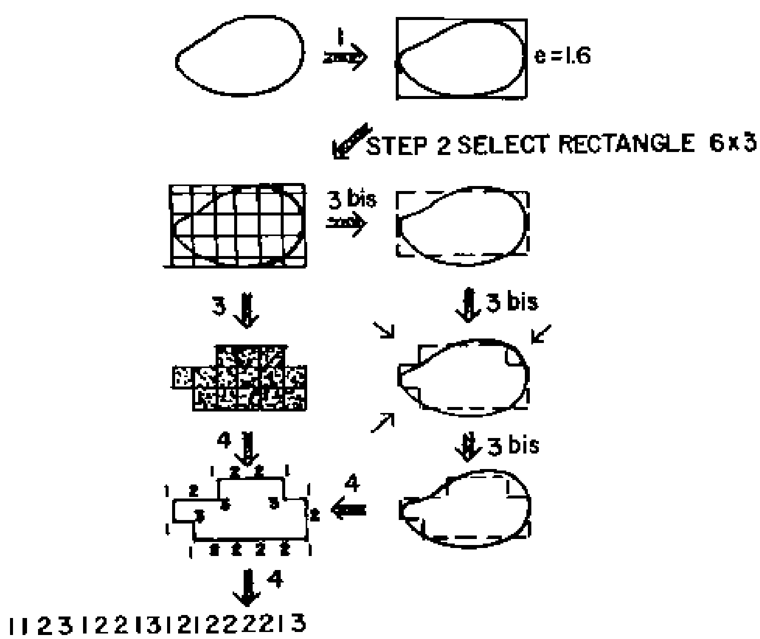


FIGURE 7 " FINDING THE SHAPE NUMBER "

The shape number of order 18 of region A is desired. The answer is 112312213121222213 The main procedure is through steps 1,2,3,4. Step 3bis is a (long) step that can be taken instead of step 3. Se text.
Each figure carries its own shape number "within it"

Degenerate regions. If the grid size is too large for some parts of a region, there will be totally blank squares that break the discrete shape into two or more pieces. Then the shape number of that region does not exist for that order. This is not an anomaly, but it is giving information regarding the minimum size grid for which a shape number makes sense (Fig. 8-II).

Meaningful shape order. A region with a very ragged and twisted perimeter will "demand" a higher order for a proper description than a region with smooth boundary;

it expects more accuracy, because of its higher information content. Bribiesca (et al,1978) measures this appropriateness, also related to degenerate regions.

THE DEGREE OF SIMILARITY BETWEEN THE SHAPES OF TWO REGIONS

The shape number of a region enables us to find out instances of a given shape, even when distorted by enlargement or rotation. It answers the question "Have these two regions the same shape?", up to an order o.

In practice, however, a shape rarely repeats itself, due to noise and the allowable variations (for instance, ten sillouettes of apples have similar but not identical shapes). The relevant questions to answer are "How much different are these two forms?", "How much do these two shapes resemble each other?", "Is region A closer in shape to B, or to C?". This section gives a procedure to quantitatively answer these questions.

When the shapes of two regions A and B are compared, we can notice that the shape of order 4 of A, $s_4(a)$, is equal to 1111 (the only shape of order 4), and is therefore equal to $s_4(b)$.

Also $s_6(a) = s_6(b)$; probably $s_8(a) = s_8(b)$. It is likely that their first few shape numbers be identical. The reason is that the discrete shapes are coarse and not varied at low orders, where the "resolution" is low.

Nevertheless, most likely $s_{100}(a) \neq s_{100}(b)$, also $s_{98}(a) \neq s_{98}(b)$, etc. This is expected, because, due to the finer precision at higher orders, there exists a large variety of shapes, thus the discrimination between A and B is more demanding.

Of course, if A and B were very similar (but not identical), one could need to go up to say 170 to find that $s_{170}(a) \neq s_{170}(b)$. On the other hand, if they are visibly different (not alike at all), already at order 10 we will be having $s_{10}(a) \neq s_{10}(b)$.

Thus, as we increase the order o of the two shape numbers $s_o(a)$ and $s_o(b)$, they begin being equal but at some order they become different from that point on. How deeply they remain equal gives us an idea of the similarity between the shapes of a and b.

Degree of similarity k between the shapes of two regions a and b: It is the largest order for which their shape numbers still coincide.

That is, it is the largest m for which $s_m(a) = s_m(b)$, but $s_{m+i}(a) \neq s_{m+i}(b)$ for all i greater than 0.

That is, we have $s_4(a) = s_4(b)$, $s_6(a) = s_6(b)$, $s_8(a) = s_8(b)$, ..., $s_k(a) = s_k(b)$, $s_{k+2}(a) \neq s_{k+2}(b)$, $s_{k+4}(a) \neq s_{k+4}(b)$, ...

If a and b are regions with degree k of similarity, we write $a \approx_k b$.

Example. For the figures of Fig. 9, we have for figures a, b and c:

$s_4(a)$ = 1 1 1 1	$s_4(b)$ = 1 1 1 1	$s_4(c)$ = 1 1 1 1
$s_6(a)$ = 1 1 2 1 1 2	$s_6(b)$ = 1 1 2 1 1 2	$s_6(c)$ = 1 1 2 1 1 2
$s_8(a)$ = 11221122	$s_8(b)$ = 12121212	$s_8(c)$ = 12121212
$s_{10}(a)$= 1122211222	$s_{10}(b)$= 1131212122	$s_{10}(c)$= 1212212122
$s_{12}(a)$= 112221131213	$s_{12}(b)$= 121221221213	$s_{12}(c)$= 121222121222
$s_{14}(a)$= 11222211231132	$s_{14}(b)$= 12121312212123	$s_{14}(c)$= 11312212212213

Therefore, a and b have a degree of similarity equal to 6: $a\approx_6 b$,
a and c have a degree of similarity equal to 6, written $a\approx_6 c$,
b and c have a degree of similarity equal to 8, written $b\approx_8 c$.
This is represented both as a similarity tree (Fig. 9b) and as a similarity matrix (Fig. 9c) where other regions were also included.

The similarity matrix is symmetrical; in fact, it is easily proved that, for arbitrary regions a and b,

(1) (Thm.) The relation "a and b have degree k of similarity" (for a fixed k) is not an equivalence relation, but

(2) (Thm.) The relation "a and b have degree of similarity of at least k" (for a fixed k) is an equivalence relation.

In fact, the equivalence classes of (2) for k=10 are nine, and a canonical shape for each of them is given in Fig. 5.

Informally speaking, the size (power) of the magnifying lens that barely confuses two regions gives the degree of similarity between such regions.

We could see the whole procedure as follows: A number is associated to each one of two regions. If the numbers are equal, the regions have identical shape. If not, another pair of numbers is deduced, and so on until we find that the two numbers coincide. The number of stages needed is an indication of the dissemblance of the two shapes.

Remarks on the degree of similarity

No parsing is necessary. To find the degree of similarity between a and b, shape numbers are compared for equality. Two shape numbers of different orders are incommensurable (can not be compared, should not, need not).

Two shape numbers of the same order are either equal or different. If different, there is no need to compare "how close they are".

To find out the degree of similarity, a binary search is used: Is $s_8(a)$ equal to $s_8(b)$? Then compare at order 100 (the highest). Then at the middle. Then at the middle of the remaining valid half. And so on. A modified binary search is better[2].

Wheatstone Bridge. In this old instrument to measure the value of resistances, an amperimeter says whether a current is zero or not. But this amperimeter does not measure the resistance itself; it only says: "current is zero. Stop!" Then the value of the resistance is obtained by a formula that does not involve the current (because it is zero!). Naturally, it does not need to be a high precission amperimeter.

In our case; the degree of similarity is not given by the shape numbers comparison test. It is given by a process that uses the comparison test.

Temperature readings. If the degree of similarity between a and b is 14, and that between c and d is 28, you can conclude that c and d are closer to each other than a and b. But we can not conclude that c and d are "twice as close in shape" as a and b. This is like the temperature: a body at 80°C is not twice as hot as one at 40°C (if you do not believe it, convert them to °F, or to °K). But see suggestion 9.

Ultradistance. If we define the distance between two shapes a and b to be the inverse of their degree of similarity, then we could easily prove that this is not only a distance, but it is also an ultradistance: it obeys $d(a,c) \leq \sup(d(a,b), d(b,c))$ in addition to the less demanding condition $d(a,c) \leq d(a,b) + d(b,c)$.

Comments on this theory of shapes

Shape numbers are not invariant under (1) reflections (mirror images); (2) skewing, where the figure is distorted by changing the angle between x and y; (3) unequal expansion, that is $X'=c_1x$, $Y'=c_2y$, with $c_1 \neq c_2$. This transforms a circle into an ellipse.

These transformations (1)-(3) alter what could be considered the (intuitive) shape of a figure. At the end of the paper a "Theory B" of shapes is presented, where condition (3) is violated, and therefore all circles and ellipses, disregarding size, eccentricity, orientation, have the same B shape numbers.

Problems with this theory of shapes

1. Ocassional loop in the similarity tree. Due to noise or the 50% requirement for quantization, and at low orders, sometimes it is observed a transitory diver-

[2] Nadler, M., remarked at his Seminar on Pattern Recognition (IRIA, France, Feb.78) that since it costs more to compare larger orders than smaller orders, do not compare at the middle point, but move instead towards the extreme with the cheapest test by an amount proportional to the ratio of high to low costs.

gence and then convergence in the shapes of two regions, v. gr., $s_8(a) = s_8(b)$, $s_{10}(a) \neq s_{10}(b)$, $s_{12}(a) = s_{12}(b)$, $s_{14}(a) \neq s_{14}(b)$, $s_{16}(a) \neq s_{16}(b)$, ... I.e., they were already different at order 10, but they are again equal at order 12 (however, only to separate soon forever). This still gives a unique shape number for a region, but makes the definition of the degree of similarity less attractive, and the procedure to find it, unreliable.

Only loops of size 2 (such as the example given) have been found, infrequently.

A way to make these loops disappear is to ignore half of the orders, for instance those not divisible by four. Orders 4, 8, 12, 16, ... remain. All the loops of size 2 have vanished (suggestion 8b).

2. Non existent shape numbers. Shape number of order o may ocassionally not exist for a given figure, due for instance to symmetrical holes of the type of figure 8.I. This does not bother the similarity procedure, but it is a nuisance not to have that shape number. See also suggestion 8a.

3. Quantization of the eccentricity. For an object of eccentricity 1.6 (Fig. 7), what rectangle will be used as the basis for computing its shape number of order 12? Will we use the 3 by 3 square (e=1) or the 4 by 2 rectangle (e=2)? An error will be commited in any case. You have to take one or the other. There seems to be no way out of this. See suggestion 5.

We now present a theory that has none of these problems.

THEORY "B" FOR SHAPE DESCRIPTION AND SHAPE COMPARISON

To obtain this new theory, we will make some changes to the old one:

1. Force the eccentricity of any region to be equal to one, by performing an anisotropic dilation of its axis, $X' = c_1x$, $Y' = c_2y$. Now a circle and an ellipse will have the same Bshape; the Bshape of a rectangle will coincide with that of an square. As far as the <u>discrete</u> shapes, the only discrete Bshapes that now exist are those obtained from squares.

2. Do not go into depressions (Fig. 8.I) with width smaller than the size of the cell of the grid. This avoids degenerate shapes (cf. also 'Reasonable shape numbers' above). That is, if a region is "scratched" by thin lines (thiner than the size of the grid) that belong to the background, either ignore them (act as if they were not there) or else, if they can not be ignored, this Theory "B" says that the size of this grid is inappropriate to describe such region, and that its Bshape number at this order does not exist.

3. Let these depressions (Fig. 8.I) generate Bshape numbers having a number of ternary digits larger than the expected order. That is, do not correct the anomaly

that these depressions cause. The perimeter of the Bshape does not tell anymore its order.

4. Elliminate the orders that are not powers of two. The only valid orders for Bshape numbers are 4, 8, 16, ... These numbers still indicate the number of sticks to place around the basic square (remember, now a rectangle is converted first into a square) of the region (refer to step 3b of Fig. 7).

The procedure is the following:

To find the Bshape number of order $o = 2^n$ of a region:

1. Find out the basic rectangle of the region and convert it into a square.

Declare that the Bshape number does not exist if the region has parts (necks, straights) or depressions (channels) narrower than 2^{2-n} or 4/o.

2. Make a grid by dividing the side of the basic square into o/4 parts.

3. Mark with a 1 each cell of the grid of step 2 that is more than 50% contained in the region (step 3bis given above could also have been used instead of this step 3). The collection of grid squares containning a 1 forms a discrete Bshape.

4. Find the shape number of the discrete Bshape of step 3, and give that as answer, even if it has more than o ternary digits.

The order of a Bshape number is four times the number of parts into which the side of the basic square was divided. It is also the perimeter (measured by the number of sticks) of the basic square.

It is no longer the number of ternary digits of the Bshape number, nor the perimeter of the discrete Bshape.

The degree of similarity between the Bshapes of two regions is obtained as before. Definition unchanged.

Downwards constructability. Given the Bshape number of order o of a region, the Bshape number of order o/2 can be deduced from it, by regroupping appropriate sets of four neighboring cells into a cell for the lower order. Therefore, if two regions have the same Bshape number of order o, they will continue to have equal numbers of smaller order, until they cease to exist. This gets rid of problem 1 'ocassional loops in the similarity tree' of the former theory.

Upwards existence. If the Bshape number of order o of a region exists, the existence of numbers for higher orders is guaranteed: (1) the inexistence of channels or isthmus of the region thiner than 4/o implies the inexistance of those narrower than 4/(o+i), for i>0; and (2) wider depressions (wider than Fig. 8.I) will produce valid parts of the Bshape number, although its number of digits may increase. This defeats problem 2 of the former theory, "non existent shape numbers".

Finally, problem 3 of the former theory "quantization of the eccentricity" is not present in Theory "B" because all eccentricities are now equal to 1.

Nevertheless, the authors like more the former theory.

Disadvantage of Theory "B". Squeezing along one axis is now a valid (Bshape preserving) transformation. Thus, either your application does not care for the eccentricity or aspect ratio, or you carry it as another parameter, in addition to the Bshape number. I suppose you are going to be carrying other parameters of the region (length, orientation) anyway.

Also, more care needs to be exercised now when selecting the major and minor axis, to avoid noise perturbations (cf. suggestion 7).

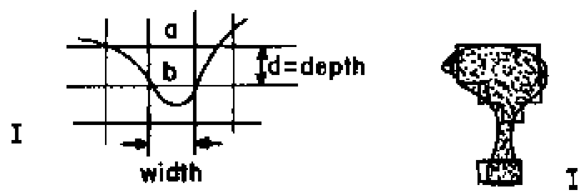
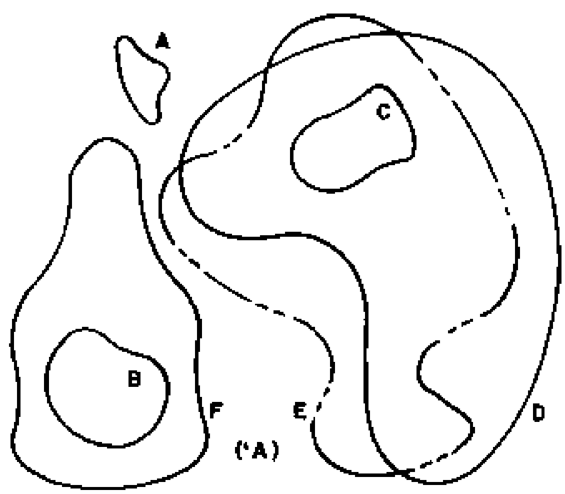
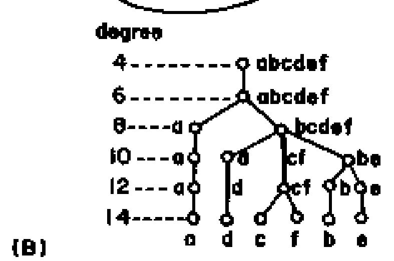


FIGURE 8 "HOLES AND DEGENERATE SHAPES"

I: A depression of depth d increases the shape number by 2d.
II: Degenerate regions split the discrete shape but do not have a shape number at this order.





	A	B	C	D	E	F
A	∞	6	6	6	6	6
B	6	∞	8	8	10	8
C	6	8	∞	8	8	12
D	6	8	8	∞	8	8
E	6	10	8	8	∞	8
F	6	8	12	8	8	∞

(C)

FIGURE 9 "DEGREE OF SIMILARITY"

(A) regions to be analyzed (B) Similarity tree for (A).
(C) Similarity matrix for regions (A).
The shapes form a hierarchy, a tree with root at degree = 4.

Suggestions and recommendations for further work

1. Use other tessellations (triangles, hexagons) instead of the square grid. I would like to see the triangle and circle as primitive shapes at low orders.

2. Use eight directions for the sticks, not four. This will produce more shape numbers of a given order, thus making the tables of figures 4-6 larger. But this is safe because the deduction of the shape number does not involve table lookup or comparison with these canonical shapes.

3. Apply these theories to Scene Analysis of coloring books (Guzmán,1971); chromosomes; silouettes of industrial parts on a conveyor belt; hand printed digits and zip codes; automatic taxonomy of shapes of shoes, airplanes, insects (their outline); texture description where the pictures are binary.

4. Extend these theories to shapes with holes inside them.

5. (Refer to problem 3 of the first theory and to step 2 of the procedure to find the shape number) a) Distort slightly the basic rectangle of the region, together with the region, so as to have it coincide exactly with the rectangle chosen in step 2: the grid is now of rectangles that are almost squares. b) Select in step 2 the rectangle of order o that minimizes the discrepancy between the areas of the region and of the rectangle.

6. Write procedure to find the eccentricity from the shape number. Hint: find the basic rectangle.

7. A better method to encase the region into a box is needed. Noise could introduce errors in length and position. Use the methods in (Bribiesca et al.,1978; Freeman and Shapira,1975; Guzmán,1971).

8. (Refer to problems 1 and 2 of first theory): a) Of course, given an order (30, say) it is possible to find the best shape number of order 30 that fits the region, by comparing (in the least squares sense) the region with all the shapes of order 30. In this way the existence of a shape number for any order and any region could be guaranteed (Bribiesca and Guzmán,1978). I suggest to look for a procedure that avoids many comparisons but still gives back the shape number of order 30. This undiscovered method could be slower, since it will be used only when the normal procedure fails. b) In order to make the loops vanish, do not use all orders. Even more, space them non linearly: use only orders 4, 6, 10, 16, 24, ...

9. Apply these theories to clustering. Do you want to group 200 figures into 24 classes according to their shapes? Construct their similarity tree, and cut it at a level such that the number of nodes at that level is approx. 24. You could answer relative likeness questions such as: "Is the difference between a and d

larger than the difference between c and f?" The answer could be: "Yes, because $a \approx_{10} d$ and $e \approx_{14} f$". e and f went together longer. They needed a stronger lens, of order 16, to separate them.

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