A HYBRID KALMAN FILTER-FUZZY LOGIC MULTISENSOR DATA FUSION ARCHITECTURE WITH FAULT TOLERANT CHARACTERISTICS

P. J. ESCAMILLA-AMBROSIO, N. MORT
Department of Automatic Control and Systems Engineering
University of Sheffield
Mappin Street, Sheffield S1 3JD, United Kingdom
Phone: +44 (0) 114 222 5619, Fax: +44 (0) 114 222 5661

Abstract- *In this work a novel Multi-Sensor Data Fusion (MSDF) architecture with fault tolerant characteristics is proposed. This MSDF architecture is based on Kalman filtering and fuzzy logic techniques. First, the measurement coming from each sensor is fed to a fuzzy–adapted Kalman filter (FKF). The adaptation is in the sense of adjusting the measurement noise covariance matrix R using a fuzzy inference system (FIS) based on a covariance matching technique. Second, another FIS, here called a fuzzy logic observer (FLO), is used to monitor the performance of each FKF. The FLO assigns a degree of confidence to each one of the FKFs. The degree of confidence indicates to what level each FKF output reflects the true value of the parameter being measured. At this level transient sensor faults are eliminated. Finally, the fused estimated measurement is obtained through a defuzzification process based on these confidence values. At this level persistent sensor faults are eliminated using a voting scheme. To demonstrate the effectiveness and accuracy of this hybrid MSDF architecture, an example with four noisy and faulty sensors is outlined. The results show very good performance.*

Key words: Multi-sensor data fusion, fault tolerant systems, fuzzy logic, adaptive Kalman filtering.

1. Introduction

The Multi-Sensor Data Fusion (MSDF) approach is described as the acquisition, processing, and synergistic combination of information gathered by various knowledge sources and sensors to provide a better understanding of a phenomenon under consideration [1]. Different MSDF techniques have been explored recently. These techniques vary from those based on well-established Kalman filtering methods [2, 3] to those based on more recent ideas coming from soft computing technology [4, 5]. However, little work has been done in exploring hybrid architectures that consider both these approaches. In this work a novel MSDF architecture with fault tolerant characteristics is proposed. This architecture is based on a hybrid structure integrating fuzzy inference systems and Kalman filtering techniques.

The general idea explored here is the combination of the advantages that both techniques have. On the one hand, Kalman filtering is recognised as one of the most powerful traditional techniques of estimation [6]. On the other hand, the main advantages derived from the use of fuzzy logic techniques, with respect to traditional schemes, are the simplicity of the approach, the capability of fuzzy systems to deal with imprecise information, and the possibility of including heuristic knowledge about the phenomenon under consideration.

The remainder of this paper is organised as follows. Section 2 describes the Kalman filtering technique and summarises the proposed approach of fuzzy-adapted Kalman filtering [7]. Section 3 describes the proposed new hybrid MSDF architecture. To show the effectiveness of this approach and its fault tolerant characteristics, in section 4 an illustrative example with four noisy and faulty sensors is outlined and results are discussed. Finally, the conclusions of this work are given in section 5.

2. Kalman filtering

The Kalman filter is an optimal recursive data processing algorithm [6] that provides a linear, unbiased, and minimum error variance estimate of the unknown state vector $x_k \in \Re^n$ at each instant $k = 1,2,\ldots,$ (indexed by the subscripts) of a discrete-time controlled process described by the linear stochastic difference equations:

$$x_{k+1} = A_k x_k + B_k u_k + w_k \qquad (1)$$
$$z_k = H_k x_k + v_k \qquad (2),$$

where x_k is an $n \times 1$ system state vector, A_k is an $n \times n$ transition matrix, u_k is an $l \times 1$ vector of the input forcing function, B_k is an $n \times l$ matrix, w_k is an $n \times 1$ process noise vector, z_k is a $m \times 1$ measurement vector, H_k is a $m \times n$ measurement matrix, and v_k is a $m \times 1$ measurement noise vector.

Both w_k and v_k are assumed to be uncorrelated zero-mean Gaussian white noise sequences with covariances:

$$E\{w_k w_i^T\} = \begin{cases} Q_k, & i = k \\ 0 & i \neq k \end{cases}, E\{v_k v_i^T\} = \begin{cases} R_k, & i = k \\ 0 & i \neq k \end{cases}$$

$$E\{w_k v_i^T\} = 0 \qquad \text{for all } k \text{ and } i \qquad (3),$$

where $E\{\cdot\}$ is the statistical expectation, superscript T denotes transpose, Q_k is the process noise covariance matrix, and R_k is the measurement noise covariance matrix.

The Kalman filter algorithm [8, 9] is organised in two groups of equations:

i) Time update (or prediction) equations:

$$\hat{x}_{k+1}^- = A_k \hat{x}_k + B_k u_k \qquad (4)$$

$$P_{k+1}^- = A_k P_k A_k^T + Q_k \qquad (5).$$

These equations project, from time step k to step $k+1$, the current state and error covariance estimates to obtain the *a priori* (indicated by the super minus) estimates for the next time step.

ii) Measurement update (or correction) equations:

$$K_k = P_k^- H_k^T [H_k P_k^- H_k^T + R_k]^{-1} \qquad (6)$$

$$\hat{x}_k = \hat{x}_k^- + K_k [z_k - H_k \hat{x}_k^-] \qquad (7)$$

$$P_k = [I - K_k H_K] P_k^- \qquad (8).$$

These equations incorporate a new measurement into the *a priori* estimate to obtain an improved *a posteriori* estimate.

2.1. Fuzzy-adapted Kalman filtering

Assuming that the process noise covariance matrix Q is known, here an innovation-based adaptive estimation (IAE) algorithm [7, 12] to adapt the measurement noise covariance matrix R and prevent filters divergence [10, 11] has been derived. In particular, the technique known as covariance-matching is used [13]. The basic idea behind this technique is to make the actual value of the covariance of the residual consistent with its theoretical value. This is done in two steps; first, the innovation sequence or residual r is defined by:

$$r_k = (z_k - H_k \hat{x}_k^-) \qquad (9),$$

and its theoretical covariance is defined by,

$$S_k = H_k P_k^- H_k^T + R_k \qquad (10),$$

obtained from the Kalman filter algorithm. Second, if it is found that the actual covariance of r_k has a discrepancy with its theoretical value, then adjustments are made to R in order to correct this mismatch.

Given the availability of the innovation sequence r_k, its actual covariance $\hat{C}_r$ is approximated by its sample covariance [12] through averaging inside a moving estimation window of size N,

$$\hat{C}_{rk} = \frac{1}{N} \sum_{i=i_0}^{N} r_i r_i^T \qquad (11),$$

where $i_0 = k - N + 1$ is the first sample inside the estimation window. The window size N is chosen empirically to give some statistical smoothing.

Now, a new variable called the Degree of Matching (*DoM*), is defined to detect the discrepancy between S and $\hat{C}_r$. This is:

$$DoM_k = S_k - \hat{C}_{rk} \qquad (12).$$

The basic idea of adaptation used by a Fuzzy Inference System (FIS) to derive an adjustment of R is as follows. It can be noted from Eq. 10 that an increment in R will increment S, and vice versa. Thus, R can be used to vary S in accordance with the value of *DoM* in order to reduce the discrepancies between S and $\hat{C}_r$. From here three general rules of adaptation are defined:

1. If $DoM \cong 0$ (this means S and $\hat{C}_r$ match almost perfectly) then maintain R unchanged.
2. If $DoM > 0$ (this means S is greater than its actual value $\hat{C}_r$) then decrease R.
3. If $DoM < 0$ (this means S is smaller than its actual value $\hat{C}_r$) then increase R.

Thus R is adjusted in this way:

$$R_k = R_{k-1} + \Delta R_k \qquad (13),$$

where ΔR is the factor that is added or subtracted from R at each instant of time. ΔR is the FIS output and *DoM* is the FIS input. A graphical representation of the Fuzzy-adapted Kalman Filter (FKF) is shown in Fig. 1.
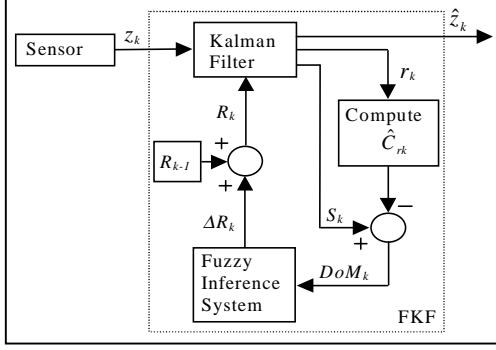
Fig. 1. Basic structure of the Fuzzy-adapted Kalman Filter (FKF).

3. Hybrid multi-sensor data fusion architecture

The objective of the proposed MSDF architecture is to obtain fused measurement data that determines the parameter being measured as precisely as possible. The main characteristic of this architecture is its fault tolerance.

In the proposed MSDF architecture it is assumed that there are n different sensors measuring the same parameter. Each sensor has its own characteristics of noise and measurement errors. The measurements obtained from these sensors are fed to a fuzzy-adapted Kalman filter (FKF), one for each sensor; thus there are n sensors and n FKFs working in parallel (see Fig. 2). A FIS, here called the Fuzzy Logic Observer (FLO), is used both to monitor the performance of each FKF and for the detection of transient sensor faults. To achieve this, the FLO was designed using three variables: the absolute value of *DoM* (*AbsDoM*), the value of *R*, and a variable called the Residual Compatibility *rC*.
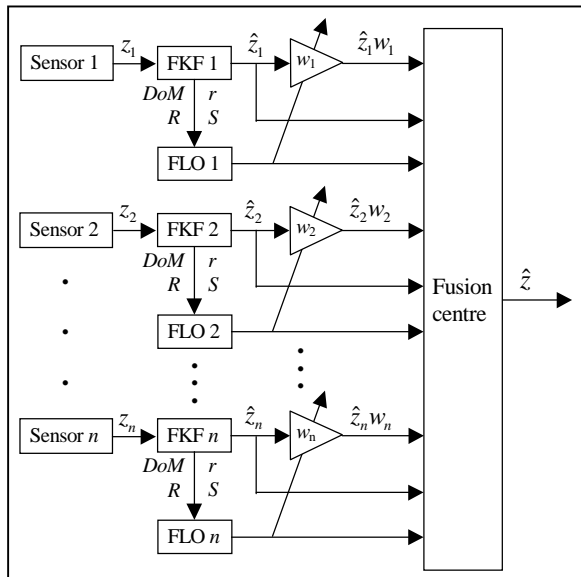

Fig. 2. New hybrid MSDF architecture.

The variable *rC* is obtained using the following relation:

$$rC_k = \frac{|r_k|}{\sqrt{S_k}} \qquad (14).$$

This value gives a measure of the actual amplitude of the residual *r* compared to its theoretical value $\sqrt{S}$. For a sensor without transient faults the value of *rC* is around 1, if this value changes abruptly, this means that a transient fault on the corresponding sensor is present.

The FLO assigns a degree of confidence *w*, a number on the interval [0, 1], to each one of the FKF (see Fig. 2). *w* indicates to what level each FKF output reflects the true value of the measurement and if this value is coming from a non-faulty sensor (transient faults). At the same time, the degree of confidence acts as a weight that indicates to the fusion centre at what confidence level it should take each FKF output value.

The fusion centre (see Fig. 2) obtains the fused estimation of the parameter being measured based on the confidence values. First of all, the fusion centre applies a voting technique to investigate if there are persistent sensor faults. If one of the estimated measurements differs markedly from the others this means a persistent sensor fault is present in that sensor, thus that value is not considered for fusion purposes. Finally a defuzzification process is applied to obtain the fused estimation of the parameter being measured. Fig. 2 shows a graphical representation of the proposed MSDF architecture.

4. Illustrative example

Consider the following linear system, which is a modified version of a tracking model [14, 15],

$$\begin{bmatrix} x_{k+1}^1 \\ x_{k+1}^2 \\ x_{k+1}^3 \end{bmatrix} = \begin{bmatrix} 0.77 & 0.20 & 0.00 \\ 0.25 & 0.75 & 0.25 \\ 0.05 & 0.00 & 0.75 \end{bmatrix} \begin{bmatrix} x_k^1 \\ x_k^2 \\ x_k^3 \end{bmatrix} + \begin{bmatrix} w_k^1 \\ w_k^2 \\ w_k^3 \end{bmatrix} \qquad (15a)$$

$$z_k = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k^1 \\ x_k^2 \\ x_k^3 \end{bmatrix} + v_k \qquad (15b),$$

with initial conditions $\hat{x}_0 = 0$, $P_0 = 0.01 I_3$, where x^1, x^2, and x^3 are the position, velocity and acceleration, respectively, of a flying object. In Eq. 15, the system noise sequence $\{w_k\}$ is uncorrelated zero-mean Gaussian noise with $Q = 0.02 I_3$.

MATLAB code was developed to simulate the process described by Eq. 15 and the proposed MSDF architecture considering four sensors measuring the position of the flying object. The simulation was carried out for 500s with a sample time of 0.5s. Q was fixed as $0.02I_3$. The actual value of R for each sensor was assumed unknown. The starting value of R in all sensors was selected as 1. Each sensor measurement is corrupted by the noise profiles described in Fig. 3. Additionally, persistent faults are introduced on sensor 1 at time ranges [100-200], [300-310], and [400-410]. During these ranges the measurement of sensor 1 is fixed as −15. Transient faults are introduced on sensor 4 at times 150, 250, 350 and 450. At these times the measurement of sensor 4 is fixed as −20.
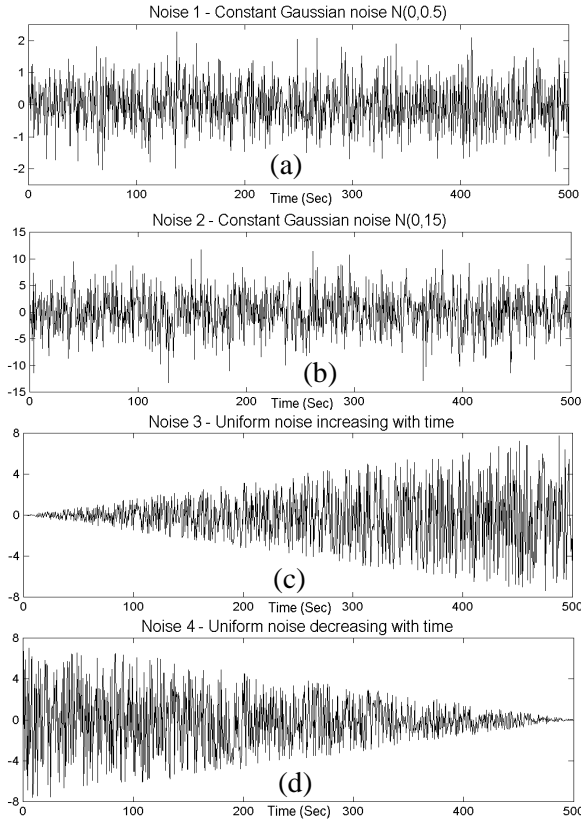

Fig. 3. (a) Noise on sensor 1, (b) noise on sensor 2, (c) noise on sensor 3, (d) noise on sensor 4.

In subsequent sections, the implementation of each one of the components of the hybrid MSDF architecture is described.

4.1 Fuzzy-adapted Kalman Filter (FKF)

Each FKF was implemented considering five fuzzy sets for *DoM*: *NM* = Negative Medium, *NS* = Negative Small, *ZE* = Zero, *PS* = Positive Small, and *PM* = Positive Medium; and five for

ΔR: *IL* = Increase Large, *I* = Increase, *M* = Maintain, *D* = Decrease, and *DL* = Decrease Large. The membership functions for *DoM* and ΔR are presented in Fig. 4.
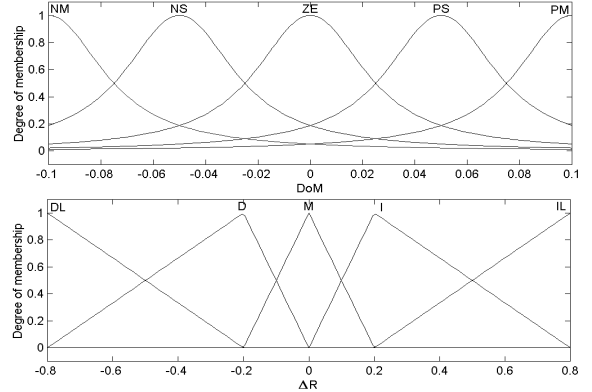

Fig. 4. Membership functions for *DoM* and ΔR.

Following the general guidelines given in section 2.1 five fuzzy rules have been formulated in order to adjust R, these are:

1. If *DoM* = *NM*, then ΔR = *IL*
2. If *DoM* = *NS*, then ΔR = *I*
3. If *DoM* = *ZE*, then ΔR = *M*
4. If *DoM* = *PS*, then ΔR = *D*
5. If *DoM* = *PM*, then ΔR = *DL*.

Thus using the compositional rule of inference sum-prod, R is adjusted as mentioned in Eq. 13. The window size N was selected empirically as 15.

4.2 Fuzzy Logic Observer (FLO)

The membership functions for *AbsDoM*, *R* and *rC* are shown in Fig. 5. Here the fuzzy labels mean: *ZE* = Zero, *S* = Small, *L* = Large, *NF* = No Fault, and *F* = Fault. For the output *w*, 3 fuzzy singletons were defined with the labels: *G*=1=Good, *AV*=0.5=Average, and *P*=0=Poor. Thus 18 rules complete the FLO rule base, as shown in Table 1.

Table 1

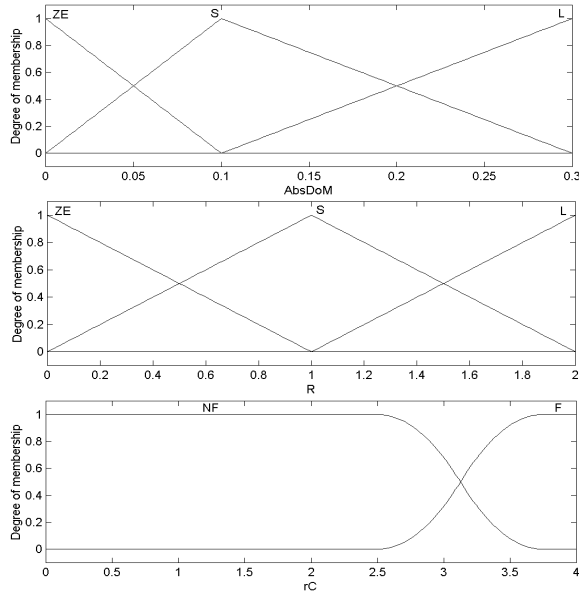|	*rC* = NF		
R / *AbsDoM*	ZE	S	L
ZE	G	G	AV
S	G	AV	P
L	AV	P	P
	rC = F		
R / *AbsDoM*	ZE	S	L
ZE	P	P	P
S	P	P	P
L	P	P	P

Fig. 5. Membership functions for *AbsDoM*, *R*, and *rC*.

The above rules are based on two simple heuristic considerations. First, if the *AbsDoM* is near to zero and *R* is near to zero and no transient fault is present, then it means the filter is working almost perfectly. Second, if one or both of *AbsDoM* and *R* increases far from zero that means the filter performance is degrading. At the same time if *rC* increases abruptly, this means that a transient fault on the corresponding sensor has been detected.

Thus, using the compositional rule of inference sum-prod and the centre of area defuzzification method, the FLO obtains a degree of confidence for each FKF.

4. 3 The fusion centre

As a first stage in the fusion centre (see Fig. 2) a voting technique [16] is used to detect persistent sensor faults. This is done by calculating the factors:

$$C_{12} = \left| \hat{z}_1 - \hat{z}_2 \right|, C_{13} = \left| \hat{z}_1 - \hat{z}_3 \right|, C_{14} = \left| \hat{z}_1 - \hat{z}_4 \right|,$$
$$C_{23} = \left| \hat{z}_2 - \hat{z}_3 \right|, C_{24} = \left| \hat{z}_2 - \hat{z}_4 \right|, C_{34} = \left| \hat{z}_3 - \hat{z}_4 \right|$$
$$\dots(16),$$

and then applying the following crisp rules:

1. If C_{12} and C_{13} and C_{14} are $\geq \alpha$ then sensor 1 has a persistent fault.
2. If C_{12} and C_{23} and C_{24} are $\geq \alpha$ then sensor 2 has a persistent fault.
3. If C_{13} and C_{23} and C_{34} are $\geq \alpha$ then sensor 3 has a persistent fault.
4. If C_{14} and C_{24} and C_{34} are $\geq \alpha$ then sensor 4 has a persistent fault.

In these rules α is a threshold factor used to indicate the degree of disagreement between each estimated measurement and the others. If one of the estimated measurements differs markedly from the others then this signal is eliminated and the corresponding sensor is assumed to be in a persistent fault state. Here α was selected as 1.2. Finally, a defuzzification process is applied to obtain the fused estimation of the parameter being measured. In this case the centre of area method is used,

$$\hat{z}_k = \frac{\sum_{i=1}^{n} \hat{z}_{ki} w_{ki}}{\sum_{i=1}^{n} w_{ki}} \qquad (17),$$

where $\hat{z}_{ki}$ is the output of the *i*-th FKF (i=1,2,3,4) and w_{ki} is its respective degree of confidence at instant of time *k*. Thus, each FKF output is weighted according to its corresponding degree of confidence *w*, and if it is found in the persistent fault state, then this output and its corresponding weight are not considered for fusion purposes.

In order to prevent possible conflicts, the following crisp rule was incorporated: If the sum of the degrees of confidence is equal to zero, then the fused output is simply the average of the non-faulty FKF outputs.

4. 4 Results

For comparison purposes, the following performance measures were adopted:

$$J_{zv} = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (za_k - z_k)^2} \qquad (18)$$

$$J_{ze} = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (za_k - \hat{z}_k)^2} \qquad (19),$$

where za_k is the actual value of the position; z_k is the measured position; and $\hat{z}_k$ is the estimated position at an instant of time *k*; n = No. of samples.

Table 2 shows the performance measures of each individual FKF and those obtained by the proposed MSDF architecture where the fusion of the four sensors is made for both cases: with non-faulty and faulty sensors (NFS and FS). From the analysis of the data, it is noted that the most accurate estimation of the position is obtained with the MSDF architecture for both cases NFS and FS. For the case of NFS the error in the estimation is 6.24% less compared to that

obtained with FKF 1 (for sensor 1), which has the best individual performance measure. At the same time this error is 58.86% less compared to that obtained with FKF 2 (for sensor 2) which has the worst individual performance measure. Fig. 6 shows the actual and fused estimated position and the corresponding error in the estimation for the NFS case.

Table 2

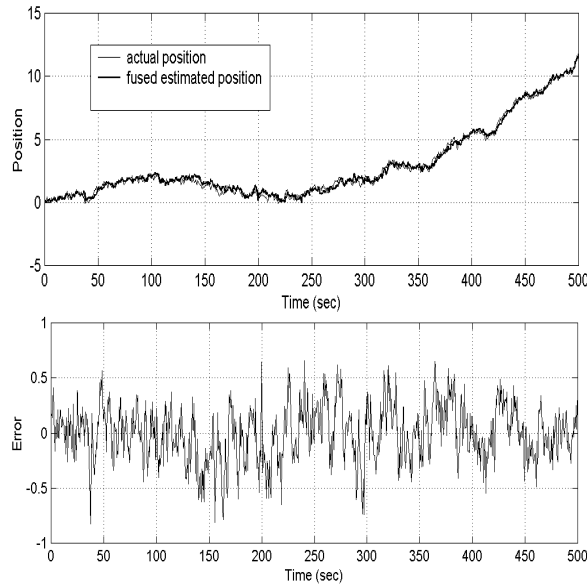| Sensor	Without faults		With faults	
	J_{zv}	J_{ze}	J_{zv}	J_{ze}
Sensor 1	0.7285	0.2693	7.8075	7.4804
Sensor 2	3.9066	0.6137	3.9066	0.6137
Sensor 3	2.5633	0.4200	2.5633	0.4200
Sensor 4	2.5928	0.3973	2.9549	0.6371
Fused		0.2525		0.2831

Fig. 6. Actual and fused estimated position and the corresponding error obtained with the MSDF architecture for non-faulty sensors.

For the case of faulty sensors the performance measure for the fused estimated position is only a little larger than that for the case previously analysed (see Table 2). The error only increases by 12.12% in spite of the faults introduced on sensor 1 and 4. Additionally, this estimation error is 32.6% smaller than that observed in the best individual FKF (in this case number 3). Thus both fused estimated measurements are more accurate than those obtained with any individual FKF. Fig. 7 shows the estimated position obtained with FKF 1; here the effects caused by the introduction of persistent sensor faults can be seen. Fig 8 shows the estimated position obtained with FKF 4; here the effects caused by the introduction of

transient sensor faults can be observed. Finally, Fig. 9 shows the actual and fused estimated position and the corresponding error for the faulty sensors case.
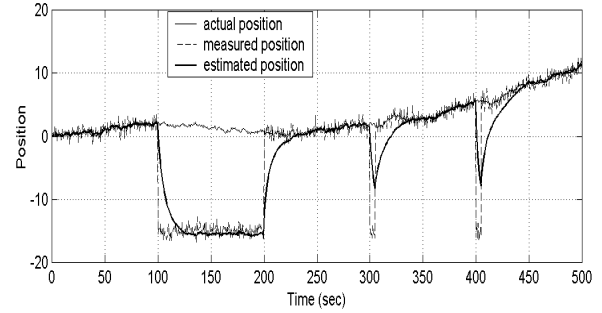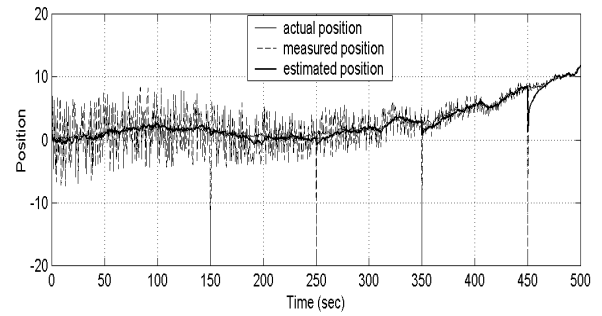

Fig. 7. Estimated position obtained with FKF 1.
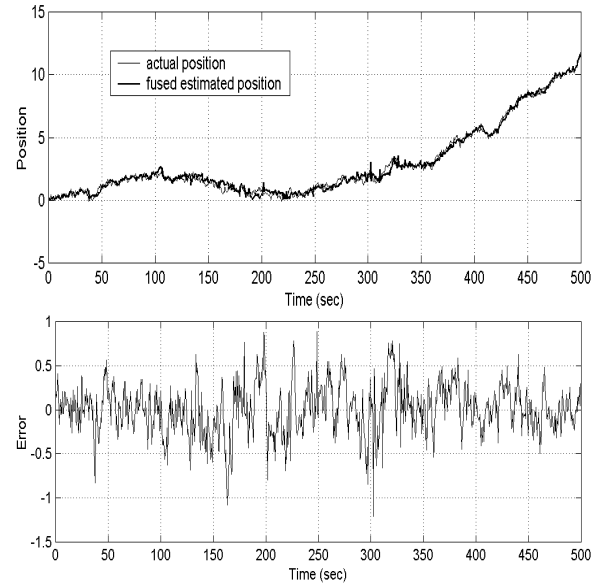

Fig 8. Estimated position obtained with FKF 4.

Fig. 9. Actual and fused estimated position and the corresponding error obtained with the MSDF architecture for faulty sensors.

5. Conclusions

A novel hybrid MSDF architecture integrating Kalman filtering and fuzzy logic techniques has been presented. The main characteristic of this architecture is its tolerance to persistent and transient sensor faults. This characteristic is

obtained by exploiting the advantages that both approaches have: the optimality of the Kalman filter and the capability of fuzzy systems to deal with imprecise information using fuzzy sets and common sense rules.

In this novel approach the linear estimations of the individual Kalman filters are improved through the adaptation of the measurement noise covariance matrix R by means of a FIS. This prevents filter divergence and relaxes the a priori assumption of the value of R. It is noteworthy that only five rules were needed to carry out this adaptation.

The role of the FLO in the proposed MSDF architecture is of great importance. This is because the fusion of the information is carried out based on the degrees of confidence generated on this element. At this level transient sensor faults are automatically removed.

The results obtained in the illustrative example are promising. They show that this novel hybrid MSDF architecture is effective in situations where there are several sensors measuring the same parameter and each sensor measurement is contaminated with a different kind of noise. This approach is also capable of tolerating transient and persistent sensor faults in an efficient way. Both fused estimated measurements (with faulty sensors and non-faulty sensors) produced better approximations to the actual value of the parameter being measured than that obtained with any single FKF. Thus the general idea of exploring the combination of traditional together with non-traditional techniques appears to be a promising avenue of investigation.

The system employed to illustrate the effectiveness of the approach presented is simple and only one parameter is considered as being measured. However the approach can be easily extended for systems with more than one parameter being measured. In fact this is the subject of current work being done by the authors.

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